Abstracts

Riccardo Adami

Lack of critical power for the Schroedinger equation with nonlinear point interaction in dimension two

Whereas in dimension one and three the dynamics induced by a nonlinear point interaction on a quantum particle mimics the behaviour of the standard NLS, the two-dimensional case proves much more involved and exotic: in particular, the blow-up phenomenon occurs at every nonlinearity power, so that the notion of critical power here does not apply. This is a joint work with R. Carlone, M. Correggi and L. Tentarelli.

Anne-Sophie Bonnet-BenDhia

A new complex frequency spectrum for the analysis of tranmission properties in perturbed waveguides

It is a common work with Lucas Chesnel and Vincent Pagneux.

We consider acoustic waveguides with a bounded cross-section, where only a finite number of modes can propagate at a given frequency, other modes being evanescent. If an infinite waveguide is locally perturbed, a superposition of incident propagating modes will generally produce a superposition of reflected propagating modes. As a consequence, only a part of the total incident energy will be transmitted from the inlet to the outlet. But at some exceptional frequencies and for particular incident waves, it may occur that 100

A first expensive way to do that is to compute the scattering matrix for all frequencies in the range of interest. We have proposed an alternative approach for which reflection-less frequencies appear directly as eigenvalues of a new (non-selfadjoint) problem. The main idea is to use a complex scaling with conjugate scaling parameters in the inlet and in the outlet. This allows to select an ingoing wave in the inlet and an outgoing wave in the outlet, which means precisely that there is no reflection, as wanted. In fact, we show that the real eigenvalues that are obtained correspond either to trapped modes or to reflection-less modes. In addition to this real spectrum, we find intrinsic complex frequencies, which also contain information about the quality of the transmission through the waveguide.

The approach can be generalized to the junction of several semi-infinite waveguides, and to other types of waves (electromagnetic waves, elastodynamic waves or water waves). Several illustrations will be presented.

Jean-Marc Bouclet

Sharp time decay estimates for dispersive equations

I will present recent results obtained with Nicolas Burq about local energy decay (and the related resolvent estimates) for the wave, Klein-Gordon and Schrödinger equations on an asymptotically Euclidean background.

Gabrielle Brüll

On the highest wave for a class of nonlocal dispersive equations

We discuss periodic traveling wave solutions for a class of *fractional KdV equations*,

$$u_t + uu_x + L_r u_x = 0, \qquad \widehat{Lu}(t,x) = |k|^{-r}\hat{u}(t,k), \qquad r > 0.$$

Of interest is the existence and regularity of highest traveling waves forming a singularity in form of a peak/cusp at its crests. Recall that the case $r = 2$ corresponds to the *reduced Ostrovsky* equation for which there exists an explicit representation of a highest, peaked traveling wave being exactly Lipschitz continuous at its crest. We would like to emphasize that, unless r is an integer, we deal with a nonlocal equation. Using a similar approach as introduced by Ehrnström & Wahlén in 2017, where the authors affirmed the conjecture that the Whitham equation admits a highest cusped wave, we prove that for **any** $r > 1$ the fractional KdV equation admits a highest peaked traveling wave, which is precisely Lipschitz continuous at its crests. We study the properties of the corresponding convolution kernel, apply bifurcation theory for the existence of nontrivial periodic traveling waves and obtain the highest wave as a limiting case at the end of a global bifurcation branch. Our analysis shows that the regularity is strongly connected to the smoothing property of the nonlocal operator L_r and we conjecture that for $r \in (0,1)$ the highest wave is cusped and merely Hölder continuous of order r. Moreover, the case $r = 1$ appears to be critical and the most challenging.
This is a joint work with R. N. Dhara.

Elek Csobo

Orbital stability of a Klein-Gordon equation with Dirac delta potentials

We study the orbital stability of standing wave solutions of a one-dimensional nonlinear Klein-Gordon equation with Dirac potentials

$$\begin{cases} u_{tt} - u_{xx} + m^2 u + \gamma\delta(x)u + i\alpha\delta(x)u_t - |u|^{p-1}u = 0, \\ u(t,x) \to 0, \quad \text{as} \quad |x| \to \infty, \\ (u(t), \partial_t u(t))|_{t=0} = (u_0, u_1), \end{cases}$$

where $\delta(x)$ is the Dirac mass at $x = 0$. The general theory to study orbital stability of Hamiltonian systems was initiated by the seminal papers of Grillakis, Shatah, and Strauss [1,2], newly revisited by De Bièvre *et. al.* in [3]. I present the Hamiltonian structure of the above system and the orbital stability properties of the standing wave solutions of the equation. A major difficulty is to determine the number of negative eigenvalues of the linearized operator around the stationary solution, which we overcome by a perturbation argument. This is a joint work with Masahito Ohta, François Genoud and Julien Royer.

[1] M. Grillakis, J. Shatah, W. Strauss, Stability theory of solitary waves in the presence of symmetry, I. *J. of Funct. Anal.* **74** (1987), no. 1, 160–197.

[2] M. Grillakis, J. Shatah, W. Strauss, Stability theory of solitary waves in the presence of symmetry, II. *J. of Funct. Anal.* **94** (1990), no. 2, 308–348.

[3] S. De Bièvre, F. Genoud, S. Rota-Nodari, Orbital stability: analysis meets geometry, in *Nonlinear optical and atomic systems,* **2146** (2015), 147–273.

Thomas Duyckaerts

Exterior energy bounds and application to the dynamics of nonlinear wave equation

In this talk I will review results on lower bounds for the exterior energy for the linear and nonlinear wave equations, and application to the classification of the dynamics of the critical nonlinear wave equation and wave maps. This concerns joint works with Hao Jia, Carlos Kenig and Frank Merle.

François Genoud

Stable solitons of the cubic-quintic NLS with a delta-function potential

This talk is about the one-dimensional nonlinear Schrdinger equation with a combination of cubic focusing and quintic defocusing nonlinearities, and an attractive delta-function potential. Physically, the model comes from nonlinear optics, and the delta-function potential models the interaction of a narrow defect with a relatively broad laser beam. I will show that all standing waves with a positive soliton profile can be determined explicitly in terms of elementary functions. I will then prove by bifurcation and spectral analysis that all these solutions are orbitally stable. A remarkable feature of the model is a regime of bistability, where two stable solitons with same wavenumber coexist. This is a joint work with Boris Malomed and Rada Weisshupl.

Anna Geyer

Stability of periodic waves in the reduced Ostrovsky equation

In this talk I will discuss the stability of smooth and peaked periodic travelling wave solutions of the reduced Ostrovsky equation, which is a model for internal waves in a rotating fluid. I will show that all smooth periodic travelling waves are spectrally stable, independent of the nonlinearity power. In the second part of the talk I will discuss our recent result on the peaked periodic waves. This is joint work with D. Pelinovsky.

Lysianne Hari

A scattering result for NLKG posed on product spaces

In this talk, we will deal with the scattering phenomenon for some nonlinear PDEs posed on a product space (or "flat waveguides") of type $\mathbb{R}^d \times \mathcal{M}^k$, the latter being a compact riemannian manifold. On one hand, this kind of results is well-known on $\mathbb{R}^d$: since one can have a good control of the nonlinear solution under some conditions on the equation, one can find an asymptotic linear state for large times. On the other hand, similar results on compact riemannian manifolds are not know to be true.

A natural question is: what happens in mixed settings ? Is it possible to obtain scattering when one only has some of the space variables in $\mathbb{R}^d$? This problem was first studied for the Schrdinger equation (NLS), but we will mostly deal with the Klein-Gordon equation (NLKG). We will see the natural conditions to have scattering in the product spaces case, and will give some ideas of proofs.

This talk is based on joint works with N. Visciglia (Pisa) and L. Forcella (Lausanne).

Romain Joly

Semi-stabilization for the damped semilinear wave equation

We consider a damped wave equation $u_{tt} + \gamma(x)u_t = \Delta u - u - f(u)$ where $f(u)u \geq 0$ and the damping γ may vanish in some parts of the domain. In the linear case $f = 0$, we know that any solution converges to zero for large time. Moreover, this decay is uniform if the geometric control condition holds. Due to the sign assumption on f, it is natural to expected that the stabilization to zero also holds when $f \neq 0$. This is a more complicated problem but several results are known.

The purpose of this talk is to study cases where the geometric control condition fails. Then, the solutions of the linear equation stabilizes to zero in a weaker way and until recently nothing was known about the nonlinear problem. In this talk we present several pioneer examples.

This is a joint work with Camille Laurent.

Zhong Wang

On stability of N-solitons of a fourth order nonlinear Schrdinger equation

In this talk, we consider a fourth order nonlinear Schrdinger equation (4NLS) which descirbes the motion of vortex filament. We show that the Hasimoto soliton of 4NLS is orbitally stable in energy space H^2. Combining the stability resut, we further establish the existence of multi-solitons of 4NLS. Finally, we investigate the dynamical stability of multi-solitons of 4NLS without employing the inverse scattering method. As a corollary, we give an alternative proof of dynamical stability of multi-solitons of NLS.